
Signal Processing with Fractals:

A Wavelet-Based Approach

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Signal Processing with Fractals: A Wavelet-Based Approach

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